Concurrency Theory

Winter 2025/26

Lecture 17: Petri Net Semantics of CCS

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https://proglang.github.io/teaching/25ws/ct.html

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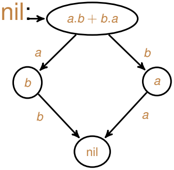
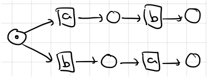
Goal: Define true concurrency semantics for (a subset of) CCS

- Distinguish between $+$ and $\|$
  - $a.b.\mathrm{nil} + b.a.\mathrm{nil}$:

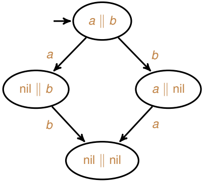


LTS

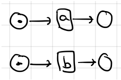


Net

  - $a.\mathrm{nil} \| b.\mathrm{nil}$:



LTS



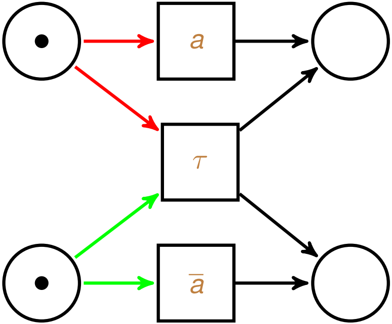
Net

- Enable analysis of CCS processes by Petri net algorithms

Non-Determinism and Unboundedness

Observations:

- Also without interleaving, parallel composition $\parallel$ can still induce non-determinism (due to conflicts), e.g., $a.\mathrm{nil} \parallel \overline{a}.\mathrm{nil}$:



- Recursive process calls can entail unboundedness, e.g.,
  $C = up.(C \parallel down.\mathrm{nil})$
  (counter; cf. Example 2.6):

The Approach

Goal: Map (a restricted class of) CCS process definitions to finite Petri nets.

Requirements:

(1) Cover as much of CCS as possible (problem: CCS is Turing complete and finite Petri nets are not).
(2) To support inductive verification proofs, nets should be constructed inductively by means of composition operators (as in CCS).

Method:

(1) Consider only guarded processes and omit restriction and relabelling operators.
(2) Specify translation $[\![.]\!] : \mathit{CCS} \to \mathit{Petri}$ in a compositional way, e.g.,

$$[\![Q_1 + Q_2]\!] := \underbrace{[\![Q_1]\!] \oplus [\![Q_2]\!]}_{\text{operation on Petri nets}}$$

CCS Revisited

Definition 17.1 (Syntax of Guarded CCS; cf. Definition 2.1)

- Let A, $\overline{A} := \{\overline{a} \mid a \in A\}$ and $Act := A \cup \overline{A} \cup \{\tau\}$ be the sets of (action) names, co-names, and actions, and let Pid be a set of process identifiers.
- The set $Prc^\dagger$ of guarded process expressions is defined by the following syntax:

$$Q ::= \sum_{i=1}^{n} \alpha_i.Q_i \quad \mid \quad Q_1 \parallel Q_2 \quad \mid \quad C$$

  where $n \in \mathbb{N}$, $\alpha_i \in Act$ and $C \in Pid$.
- Also, every process call C must be guarded, i.e., occur in an expression of the form $\alpha.Q$.
- A guarded process definition is an equation system of the form

$$(C_i = Q_i \mid 1 \leq i \leq k)$$

  where $k \geq 1$, $C_i \in Pid$ (pairwise distinct), and $Q_i \in Prc^\dagger$ (with identifiers from $\{C_1, \ldots, C_k\}$).

Notes

Petri Nets Revisited

In order to connect transitions to actions and to support the handling of process identifiers, we introduce labels for transitions and places.

Definition 17.2 (Labelled Petri net; cf. Definition 14.2)

A labelled Petri net N is a quintuple (P, T, F, l, m) where:

- P is a finite set of places,
- T is a finite set of transitions with $P \cap T = \emptyset$,
- $F \subseteq (P \times T) \cup (T \times P)$ are the arcs,
- $l : T \to Act$ is the transition labelling, and
- $m : P \dashrightarrow Pid$ is the (partial) place labelling.

Adding an initial marking $M_0 : P \to \mathbb{N}$ yields a labelled elementary system net (P, T, F, l, m, M_0).

Interleaving Semantics Revisited

Definition 17.3 (Marking graph; cf. Definition 14.18)

Let $N = (P, T, F, l, m, M_0)$ be a labelled elementary system net and $M : P \to \mathbb{N}$.

- Marking M **enables** a transition $t \in T$ if $M(p) \geq 1$ for each place $p \in {}^\bullet t$.
- Its **firing** leads to marking M', denoted by the **step** relation $M \xrightarrow{l(t)} M'$ and defined for each place $p \in P$ by

$$M'(p) := M(p) - F(p, t) + F(t, p)$$

  where we represent F by its characteristic function.
- The **marking graph** of N has as nodes the reachable markings of N and as edges the corresponding steps of N.[a]

[a] Due to transition labels, marking graphs are generally no longer **deterministic** LTSs.

Guarded Choice I

(**Reminder:** $Q ::= \sum_{i=1}^{n} \alpha_i.Q_i \mid Q_1 \parallel Q_2 \mid C \in Prc^{\dagger}$)

Approach: Implement non-determinism by conflicting transitions (one for each choice) and branch to outset of respective subnet.[1]

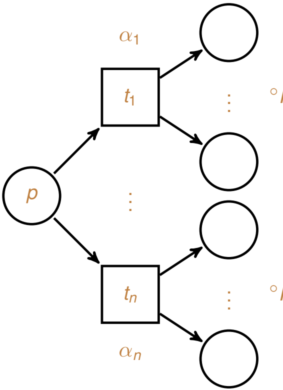
Translating guarded choice

Let $Q = \sum_{i=1}^{n} \alpha_i.Q_i \in Prc^{\dagger}$ and $[\![Q_i]\!] = N_i = (P_i, T_i, F_i, l_i, m_i)$ for $1 \leq i \leq n$. Then

$$[\![Q]\!] := (P \mathbin{\dot\cup} P', T \mathbin{\dot\cup} T', F \mathbin{\dot\cup} F', l \mathbin{\dot\cup} l', m)$$

where

$$\begin{array}{ll} P := \bigcup_{i=1}^{n} P_i & P' := \{p\} \\ T := \bigcup_{i=1}^{n} T_i & T' := \{t_1, \ldots, t_n\} \\ F := \bigcup_{i=1}^{n} F_i & F' := \{(p, t_i) \mid 1 \leq i \leq n\} \mathbin{\dot\cup} \bigcup_{i=1}^{n} \{t_i\} \times {}^{\circ}N_i \\ l := \bigcup_{i=1}^{n} l_i & l' := [t_i \mapsto \alpha_i \mid 1 \leq i \leq n] \\ m := \bigcup_{i=1}^{n} m_i & \end{array}$$



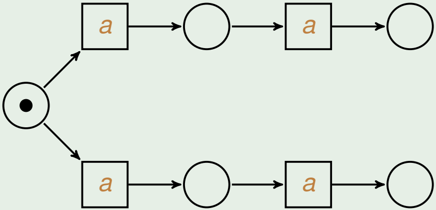
Guarded Choice II

Example 17.4

(1) $Q = \text{nil} \quad (= \sum_{\emptyset} \alpha_i.Q_i)$:

(2) $Q = a.\text{nil}$:

(3) $Q = a.b.\text{nil} + b.a.\text{nil}$:



Approach:

- Model concurrency by disjoint union of subnets, enlarged by τ-transitions for all possible synchronisation operations.
- The latter are enabled by transitions in both subnets with complementary action labels.

Translating parallel composition

Let $Q = Q_1 \parallel Q_2 \in Prc^\dagger$ and $[\![Q_i]\!] = N_i = (P_i, T_i, F_i, l_i, m_i)$ for $i \in \{1, 2\}$ (all P_i and T_i disjoint). Then

$$[\![Q]\!] := (P_1 \mathbin{\dot\cup} P_2, T_1 \mathbin{\dot\cup} T_2 \mathbin{\dot\cup} T_\tau, F_1 \mathbin{\dot\cup} F_2 \mathbin{\dot\cup} F_\tau, l_1 \mathbin{\dot\cup} l_2 \mathbin{\dot\cup} l_\tau, m_1 \mathbin{\dot\cup} m_2)$$

where

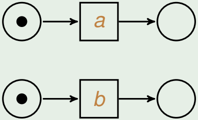
$$T_\tau := \{(t_1, t_2) \mid t_1 \in T_1, l_1(t_1) \in A \cup \overline{A}, t_2 \in T_2, l_2(t_2) = \overline{l_1(t_1)}\}$$ (new τ-transitions

$$F_\tau := \{(p_1, (t_1, t_2)), (p_2, (t_1, t_2)), ((t_1, t_2), p_1'), ((t_1, t_2), p_2') \mid (t_1, t_2) \in T_\tau, p_1 \in {}^\bullet t_1, p_2 \in {}^\bullet t_2, p_1' \in t_1^\bullet, p_2' \in t_2^\bullet\}$$ (corresponding arc

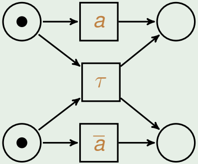
Parallel Composition II

Example 17.5

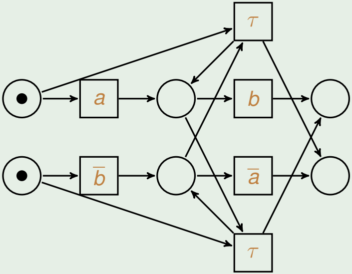
(1) $Q = a.\mathrm{nil} \parallel b.\mathrm{nil}$:



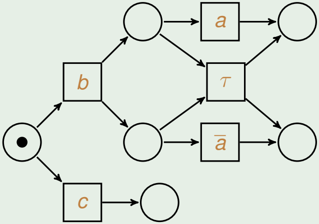
(2) $Q = a.\mathrm{nil} \parallel \overline{a}.\mathrm{nil}$:



(4) $Q = a.b.\mathrm{nil} \parallel \overline{b}.\overline{a}.\mathrm{nil}$:



(3) $Q = b.(a.\mathrm{nil} \parallel \overline{a}.\mathrm{nil}) + c.\mathrm{nil}$:



Recursive Process Calls I

Approach: Introduce labelled places for process calls (using mapping m), replace each of them by arcs to all initial places of the corresponding process expression (convert tail recursion to loop).

Translating recursive process calls

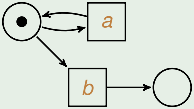
- For a process call $C \in Prc^\dagger$ ($C \in Pid$), we let
$$[\![C]\!] := (\{p\}, \emptyset, \emptyset, \emptyset, [p \mapsto C]).$$
- For a guarded process definition $D = (C_i = Q_i \mid 1 \le i \le k)$ ($C_i \in Pid$, $Q_i \in Prc^\dagger$) with $[\![Q_i]\!] = N_i = (P_i, T_i, F_i, I_i, m_i)$ for $1 \le i \le k$, we let
$$[\![Q]\!] := (P \setminus P', T, F \setminus (T \times P') \mathbin{\dot\cup} F', I, \emptyset)$$
where

$$\begin{aligned} P &:= \textstyle\bigcup_{i=1}^n P_i && \text{(all place} \\ P' &:= \textstyle\bigcup_{i=1}^n P'_i && \text{(process} \\ P'_i &:= m^{-1}(\{C_i\}) \quad (= \{p \in P \mid m(p) = C_i\}) && \text{(calls of } C \\ T &:= \textstyle\bigcup_{i=1}^n T_i && \text{(all transit} \\ F &:= \textstyle\bigcup_{i=1}^n F_i && \text{(all flows)} \end{aligned}$$

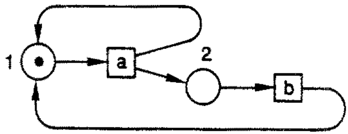
Example 17.6

(1) Call $a.C$:

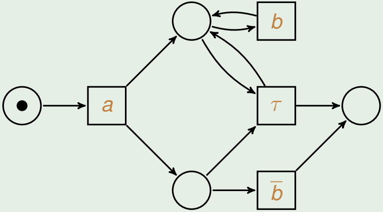
(2) Definition
$C = a.C + b.\mathrm{nil}$:



(3) Definition
$C = a.(C \parallel b.C)$:



(4) Definition
$C = a.(D \parallel \overline{b}.\mathrm{nil}),\ D = b.D$:



Correctness of Translation I

Theorem 17.7

Let $Q \in Prc^\dagger$ be a guarded process expression, and let

$$[\![Q]\!] = N = (P, T, F, l, m, M_0)$$

be its labelled elementary system net with initial marking $M_0 = {}^\circ N$.
Then $LTS(Q)$ and the marking graph of N are strongly bisimilar.

Proof.

see Ursula Goltz: *On representing CCS programs by finite Petri nets*, MFCS 1988 □

Conjecture

N is bounded iff $LTS(Q)$ is finite.

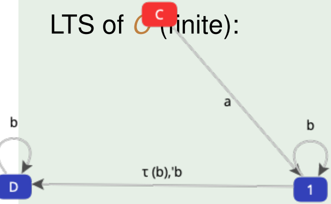
Correctness of Translation II

Example 17.8 (CCS process with finite LTS; cf. Example 17.6(4))

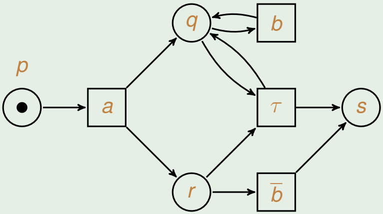
Process definition:

$$C = a.(D \parallel \overline{b}.\text{nil}), \ D = b.D$$

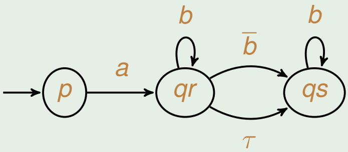
LTS of C (finite):



Net (one-bounded):



Marking graph:

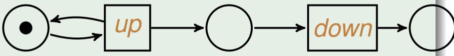


Example 17.9 (CCS process with infinite LTS; cf. Example 2.6)

Definition of counter process :

$$C = up.(C \parallel down.\mathrm{nil})$$

Net:

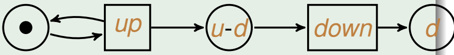


Reachable states:

$$C \xrightarrow{w} C \parallel (down.\mathrm{nil})^{u-d} \parallel \mathrm{nil}^{d}$$

where $w \in \{up, down\}^*$
with $|w|_{up} = u$ and $|w|_{down} = d$
(and $|v|_{down} \leq |v|_{up}$
for each prefix v of w).

Corresponding configurations:



Summary

- Guarded CCS processes without restriction and relabelling can be mapped to finite Petri nets.
- Interleaving/synchronisation is handled via conflicting transitions, and recursion via looping.
- The resulting marking graph is strongly bisimilar to the (LTS of) the CCS process.
- Conjecture: The net is bounded iff the LTS is finite.