Concurrency Theory

Winter 2025/26

Lecture 14: Interleaving Semantics of Petri Nets

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Programming Languages Group
University of Freiburg

https://proglang.github.io/teaching/25ws/ct.html

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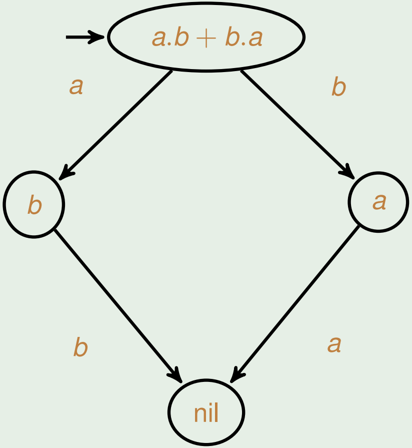
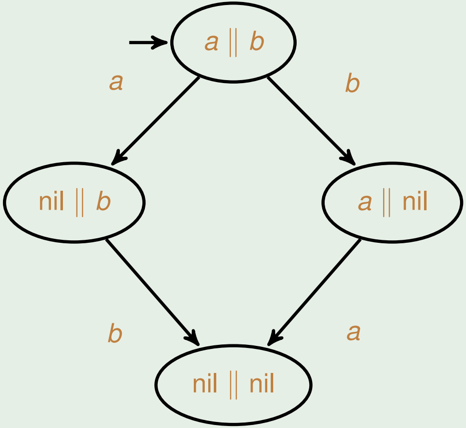
Outline of Lecture 14

Motivation

Example 14.1 (LTSs of CCS processes)



LTS of $a.b.\text{nil} + b.a.\text{nil}$



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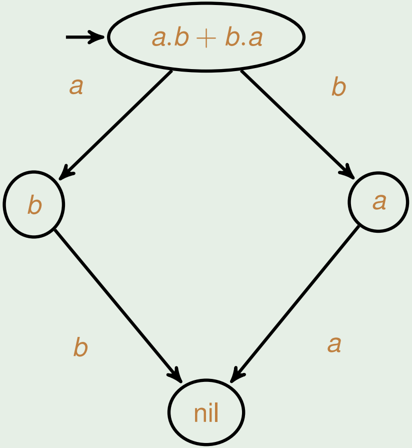

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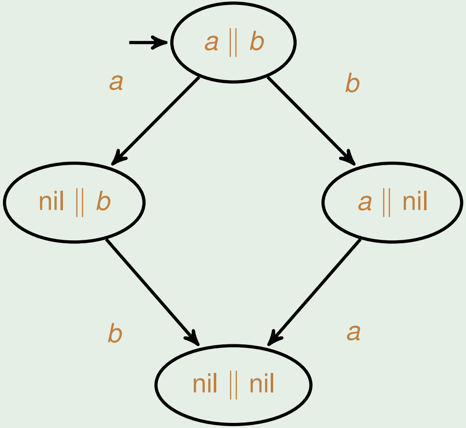

LTS of $a.\mathrm{nil} \parallel b.\mathrm{nil}$

Carl Adam Petri (1926–2010)

Semantics: Executions and Traces

Models of computation in the 1960s: lambda-calculus, finite automata, Turing machines, ...

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States: current configurations of the machine

- One or more initial states
- Possibly some distinguished final states

Transitions: moves between configurations

Lambda calculus	$(\lambda x.xx)(\lambda y.y)$	$\longrightarrow$	$(\lambda y.y)(\lambda z.z)$
Turing machine	$0010q_1011$	$\longrightarrow$	$001q_201011$
Finite automaton	q_1	$\xrightarrow{a}$	q_2
Pushdown automaton	$(q_1, XYYZ)$	$\xrightarrow{a}$	$(q_2, XYXYYZ)$

Executions: alternating sequences of states and transitions

C.A. Petri points out a discrepancy between how Theoretical Physics and Theoretical Computer Science described systems in 1962:

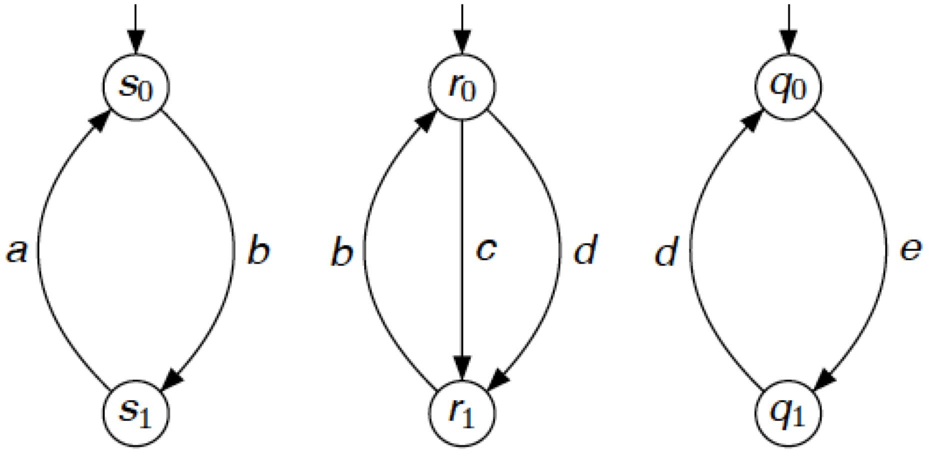
Theoretical Physics describes systems as a collection of interacting particles (subsystems), without a notion of global clock or simultaneity

Theoretical Computer Science describes systems as sequential virtual machines going through a temporally ordered sequence of global states

Petri's question:

Which kind of abstract machine should be used to describe the physical implementation of a Turing machine?

A graphical representation of interacting finite automata:



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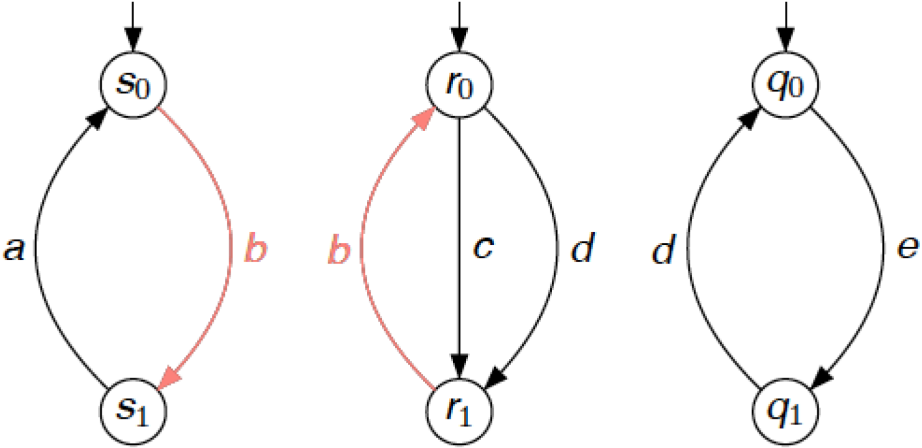

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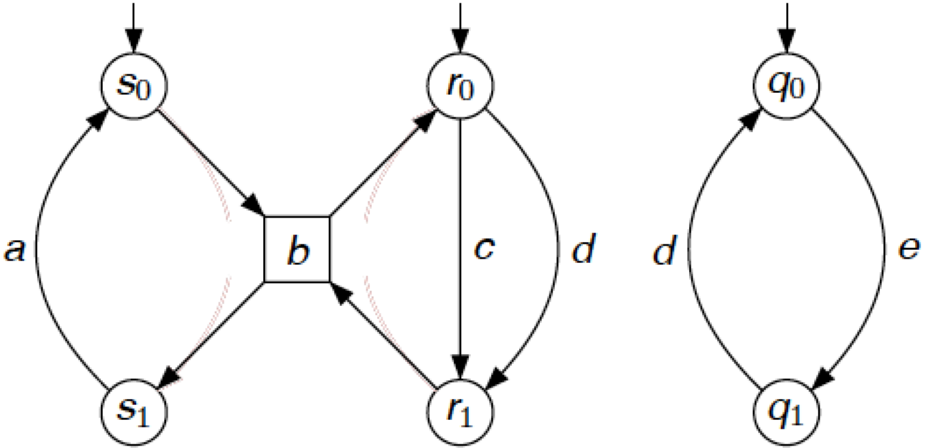

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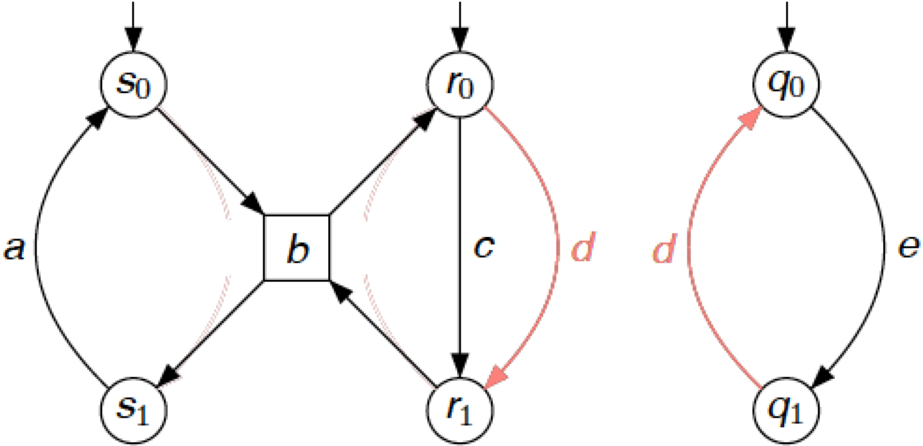

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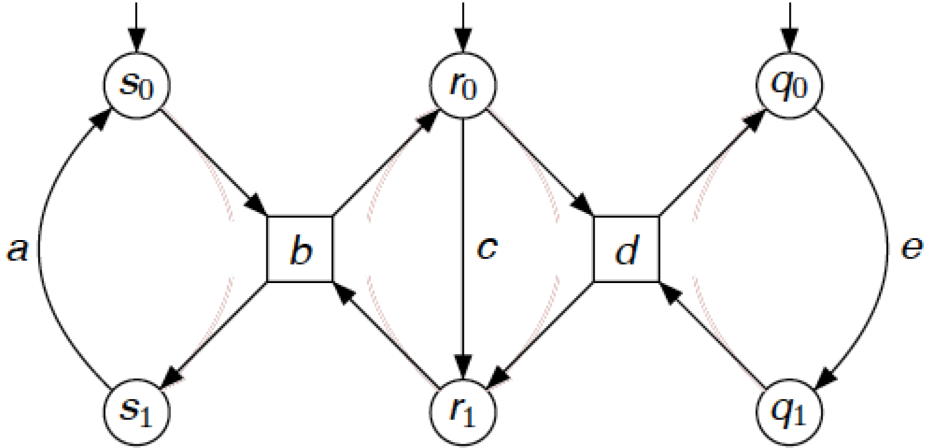

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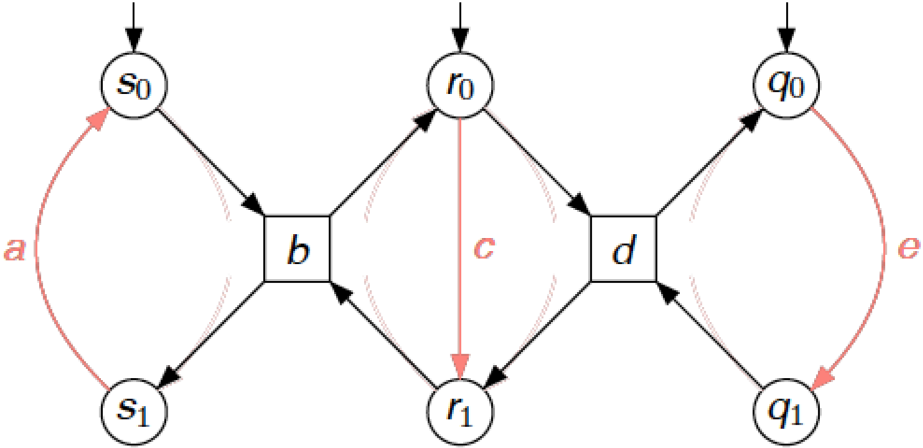

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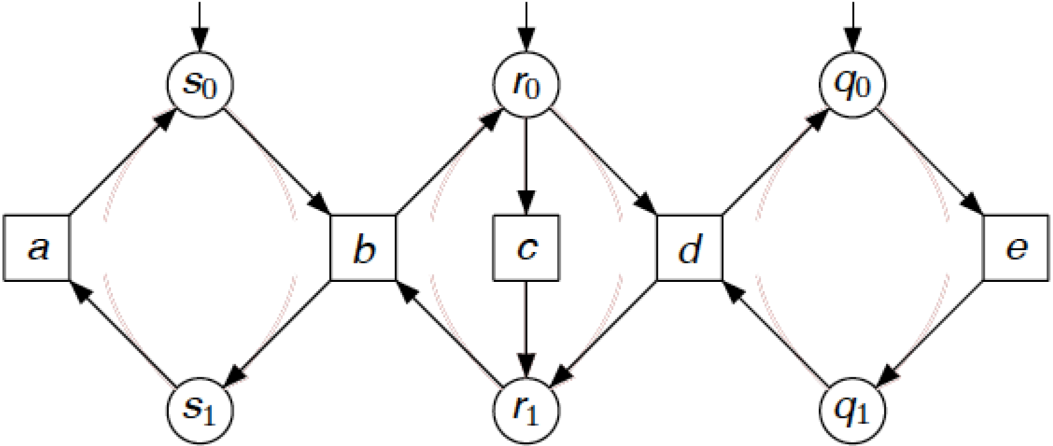

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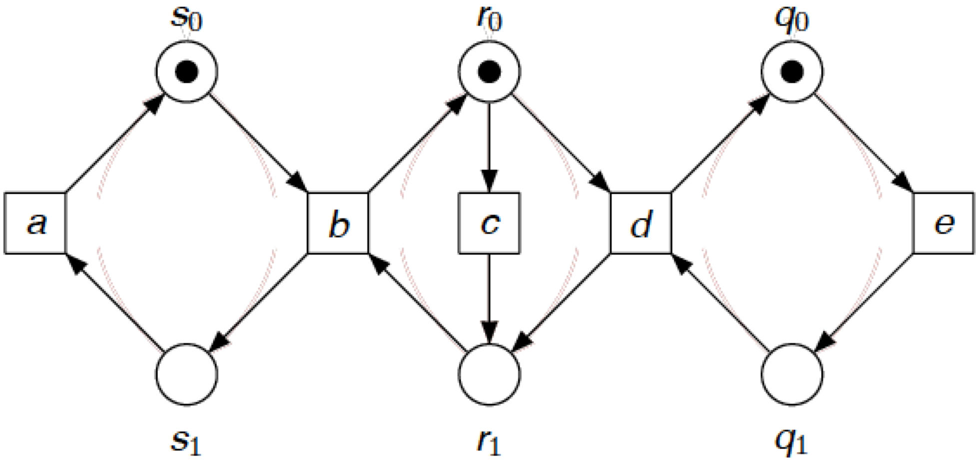

Outline of Lecture 14

Components of a Net

A Petri net is a structure with two kinds of elements: places and transitions.
These are connected by arcs.

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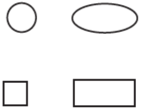

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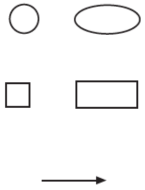
Places and transitions are connected to each other by directed arcs. Graphically, an arc is represented by an arrow. An arc models an abstract, sometimes only notional relation between components.



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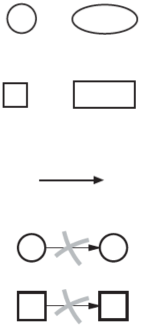

Definition 14.2 (Petri net)

A Petri net N is a triple (P, T, F) where:

- P is a finite set of places,
- T is a finite set of transitions with $P \cap T = \emptyset$, and
- $F \subseteq (P \times T) \cup (T \times P)$ are the arcs.[a]

Places and transitions are generically called nodes.

[a] F is also called the flow relation.

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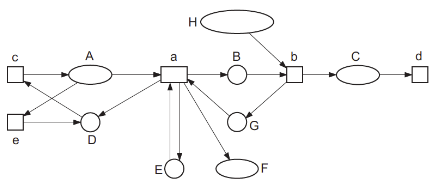
Example 14.3



$$P = \{A, B, C, \ldots\}$$
$$T = \{a, b, c, \ldots\}$$
$$F = \{(A, a), (a, B), (B, b), \ldots\}$$

Pre- and Post-Sets

Definition 14.4 (Pre- and post-sets)

Let node $x \in P \cup T$.

- The pre-set of x is defined by $\quad {}^\bullet x := \{y \mid (y, x) \in F\}$.
- The post-set of x is defined by $\quad x^\bullet = \{y \mid (x, y) \in F\}$.

Two nodes $x, y \in P \cup T$ form a loop if $x \in {}^\bullet y$ and $y \in {}^\bullet x$.

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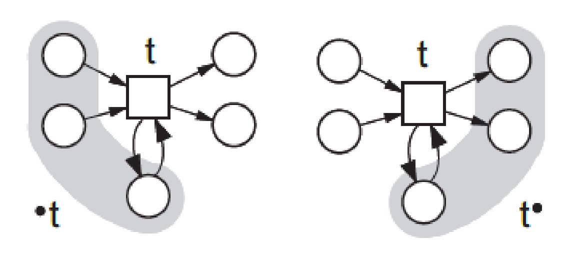

Markings

Definition 14.6 (Marking)

- A marking M of a net $N = (P, T, F)$ is a mapping $M : P \to \mathbb{N}$.
- For net $N = (P, T, F)$ and marking M_0, the quadruple (P, T, F, M_0) is called an elementary system net with initial marking M_0.

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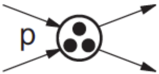
Intuition:

- A marking can be seen as a multiset of places.
- It defines a distribution of tokens across places.
- Tokens are depicted as black dots.



$M(p) = 3$

Remark: In generic (= non-elementary) system nets, several types (colours) of tokens can be distinguished.

Transition Firing

Definition 14.7 (Enabling and occurrence of a transition)

Let (P, T, F, M_0) be an elementary system net and $M : P \to \mathbb{N}$.

- Marking M enables a transition $t \in T$ if $M(p) \geq 1$ for each place $p \in {}^\bullet t$.
- Transition $t \in T$ can occur in marking M if t is enabled in M.
- Its occurrence or firing leads to marking M', denoted by the step relation $M \xrightarrow{t} M'$ and defined for each place $p \in P$ by

$$M'(p) := M(p) - F(p, t) + F(t, p)$$

where we represent relation F by its characteristic function.

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Intuition: Transition t is enabled whenever every $p \in {}^\bullet t$ holds at least one token.
On t's occurrence, one token is removed from each place in ${}^\bullet t$, and one token is put in each place in $t^\bullet$:

$$M'(p) = \begin{cases} M(p) - 1 & \text{if } p \in {}^\bullet t \text{ and } p \notin t^\bullet \\ M(p) + 1 & \text{if } p \in t^\bullet \text{ and } p \notin {}^\bullet t \\ M(p) & \text{otherwise} \end{cases}$$

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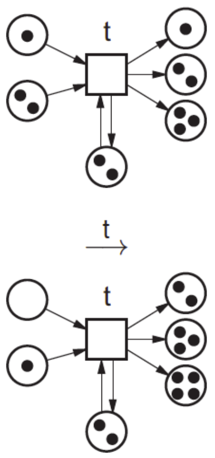


Outline of Lecture 14

The Interleaving Semantics of Petri Nets I

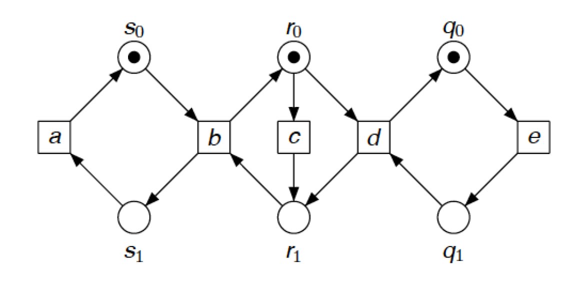
Goal: Establish an execution semantics by mapping a Petri net to a labelled transition system

States: markings (i.e., distributions of tokens over the net)

Transitions: $M \xrightarrow{t} M'$ ("steps")

Sequential runs: $M_0 \xrightarrow{t_1} M_1 \xrightarrow{t_2} M_2 \xrightarrow{t_3} \ldots$ (step sequences)

Example 14.9



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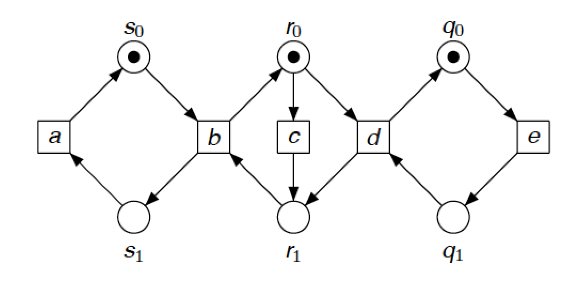
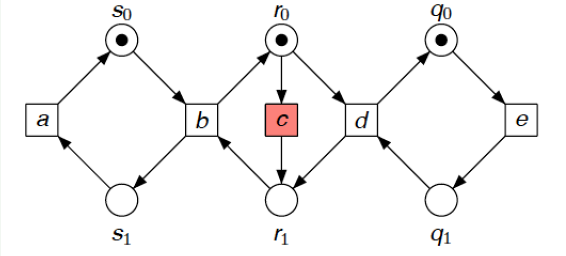


$$\begin{matrix} s_1 \\ r_1 \\ q_1 \end{matrix} \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

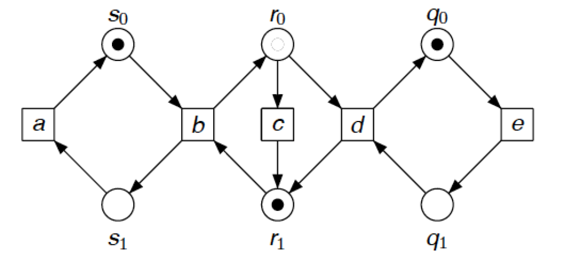
(As the marking for s_0 is the complement of s_1, the marking for s_0 is omitted. The same applies to the places r_0 and q_0.)

Example 14.9 (continued)



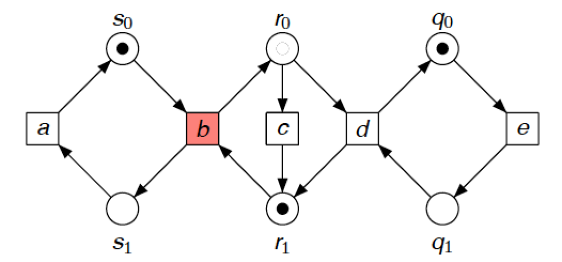
$$\begin{array}{c} s_1 \\ r_1 \\ q_1 \end{array} \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \xrightarrow{c}$$

Example 14.9 (continued)



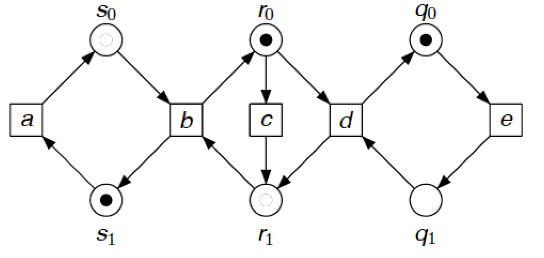
$$\begin{matrix} s_1 \\ r_1 \\ q_1 \end{matrix} \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \xrightarrow{c} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

Example 14.9 (continued)



$$\begin{matrix} s_1 \\ r_1 \\ q_1 \end{matrix} \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \xrightarrow{c} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \xrightarrow{b}$$

Example 14.9 (continued)



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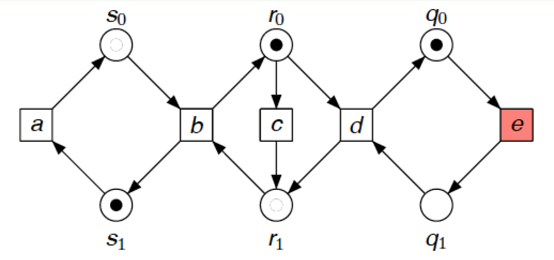

$$
\begin{matrix} s_1 \\ r_1 \\ q_1 \end{matrix}
\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}
\xrightarrow{c}
\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}
\xrightarrow{b}
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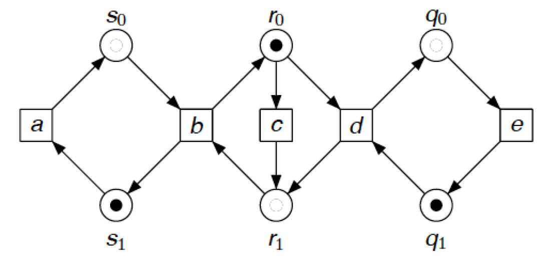
\xrightarrow{e}
$$

The Interleaving Semantics of Petri Nets VIII

Example 14.9 (continued)



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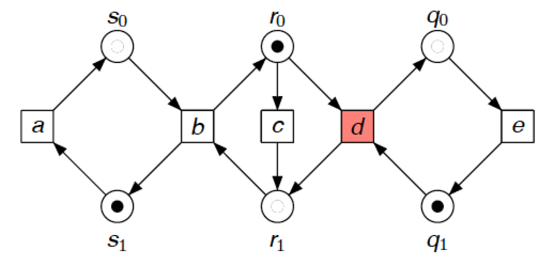

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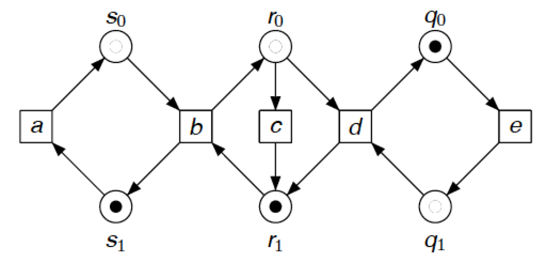

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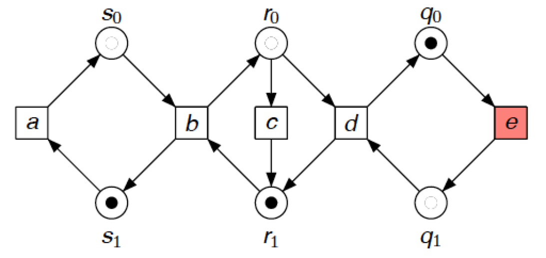

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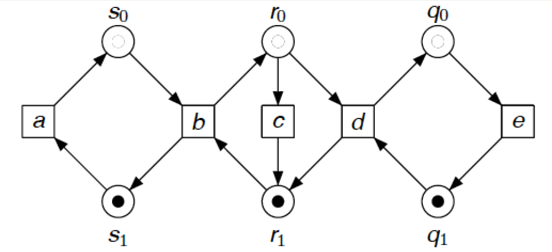
The Interleaving Semantics of Petri Nets XII

Example 14.9 (continued)



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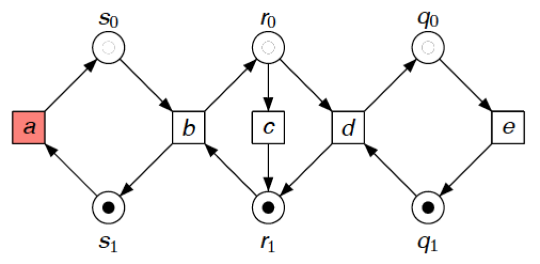

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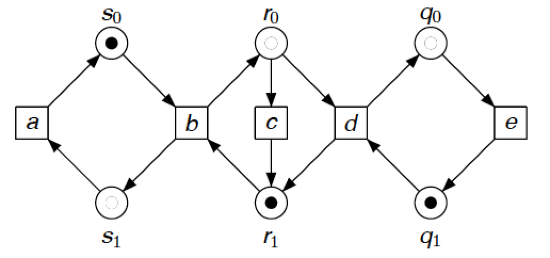
Example 14.9 (continued)



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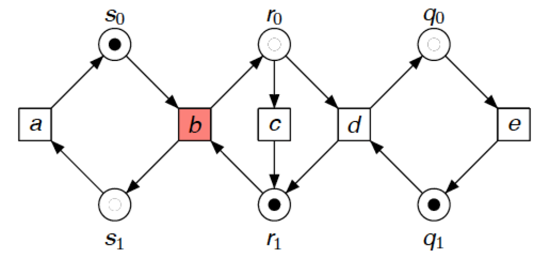
The Interleaving Semantics of Petri Nets XV

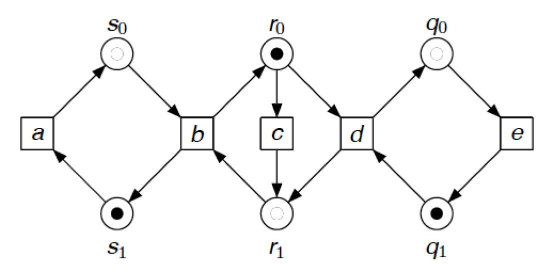
Example 14.9 (continued)



$$\begin{matrix} s_1 \\ r_1 \\ q_1 \end{matrix} \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \xrightarrow{c} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \xrightarrow{b} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \cdots \xrightarrow{e} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \xrightarrow{a} \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \xrightarrow{b}$$

The Interleaving Semantics of Petri Nets XVI

Example 14.9 (continued)



$$\begin{matrix} s_1 \\ r_1 \\ q_1 \end{matrix} \quad \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \xrightarrow{c} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \xrightarrow{b} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \cdots \xrightarrow{a} \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \xrightarrow{b} \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

Definition 14.10 (Step sequence)

Let (P, T, F, M_0) be an elementary system net.

- A sequence of transitions $\sigma = t_1\, t_2 \ldots t_n \in T^*$ is a **step sequence** if there exist markings $M_1, \ldots, M_n$ such that

$$M_0 \xrightarrow{t_1} M_1 \xrightarrow{t_2} \cdots \xrightarrow{t_{n-1}} M_{n-1} \xrightarrow{t_n} M_n.$$

Reachable Markings

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Example 14.11

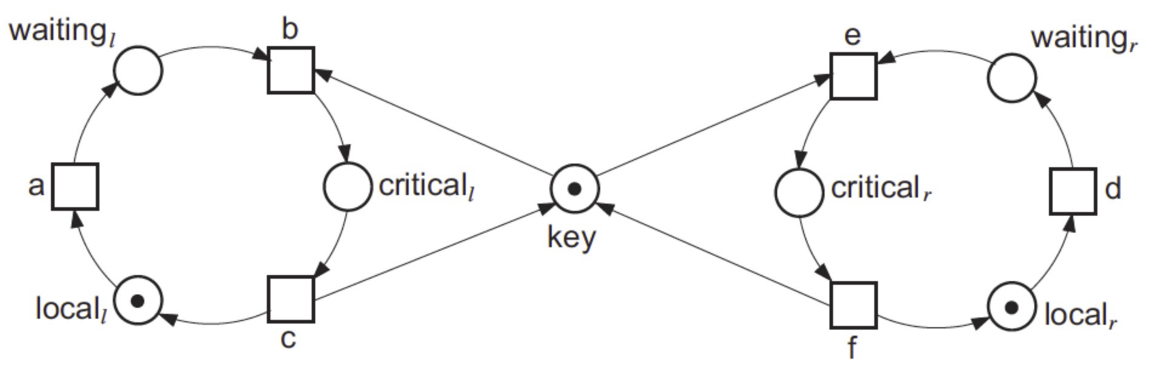
In the previous example,

$$\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \xrightarrow{cbedeab} \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

Mutual Exclusion I

Example 14.12

Two processes cycling through the states *local*, *waiting* and *critical*:



left process semaphore right process

Example 14.12

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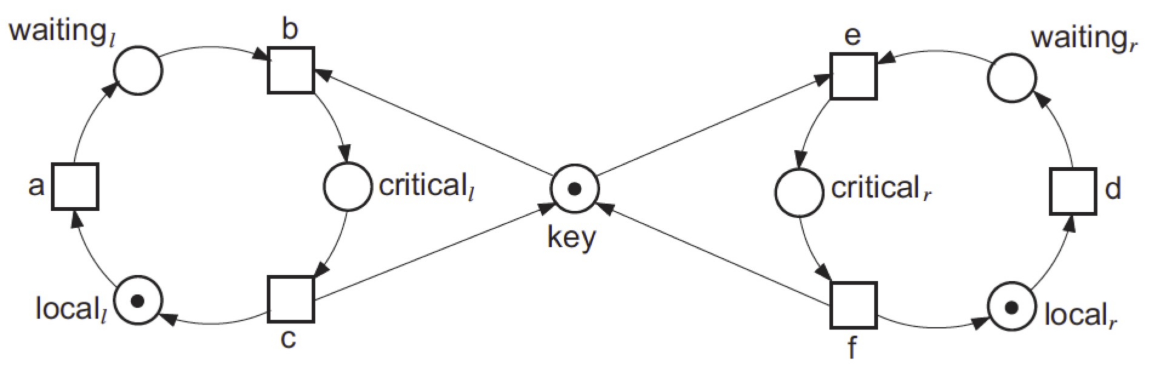
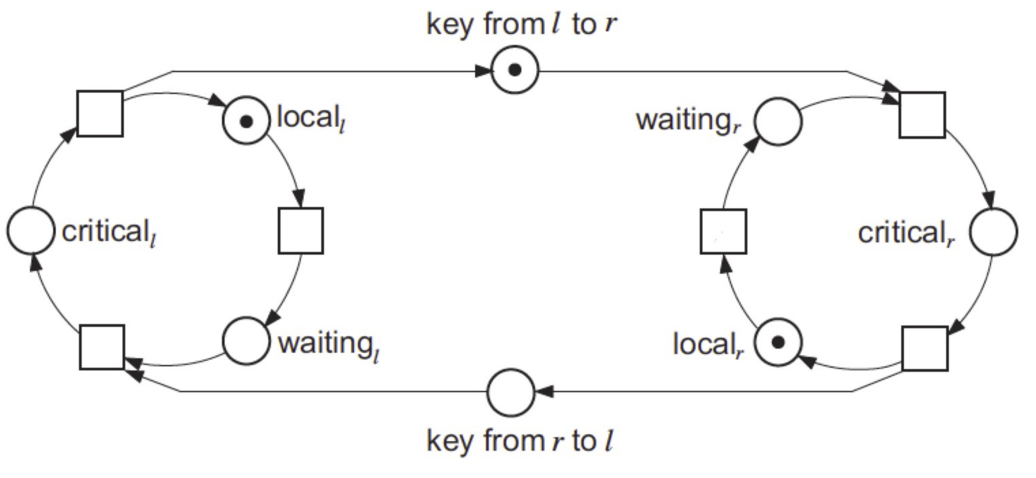


left process semaphore right process

Between transitions *b* and *e*, a conflict can arise infinitely often.
No strategy has been modelled to solve this conflict.

Example 14.13

A strategy where processes are acquiring access in an alternating fashion:



One-Bounded Elementary System Nets

Definition 14.14 (One-boundedness)

An elementary system net $N = (P, T, F, M_0)$ is called one-bounded if for each reachable marking M and place $p \in P$,

$$M(p) \leq 1.$$

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Remark: Markings of one-bounded elementary system nets can be described as a set of places.

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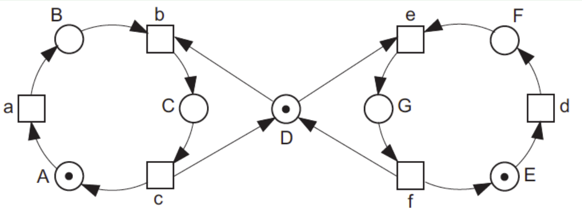
Example 14.15



Two steps beginning in marking ADE: $ADE \xrightarrow{a} BDE$ and $ADE \xrightarrow{d} ADF$.

Outline of Lecture 14

Sequential Runs

Definition 14.16 (Sequential run)

Let $N = (P, T, F, M_0)$ be an elementary system net.

- A sequential run of N is a sequence $M_0 \xrightarrow{t_1} M_1 \xrightarrow{t_2} \cdots$ of steps of N starting with the initial marking M_0.
- A run can be finite or infinite.
- A finite run $M_0 \xrightarrow{t_1} M_1 \xrightarrow{t_1} \cdots \xrightarrow{t_n} M_n$ is complete if M_n does not enable any transition.

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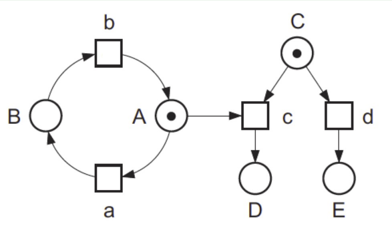
Example 14.17



A sample complete run:

$$AC \xrightarrow{a} BC \xrightarrow{b} AC \xrightarrow{c} D$$

A sample incomplete run:

$$AC \xrightarrow{d} AE \xrightarrow{a} BE$$

Marking Graph

Definition 14.18 (Marking graph)

The marking graph of a net N has as nodes the reachable markings of N and as edges the corresponding steps of N.[a]

[a] Since firing an (enabled) transition in a marking yields a unique successor marking, marking graphs are a deterministic LTS.

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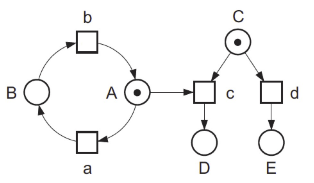
Example 14.19



A sample elementary system net

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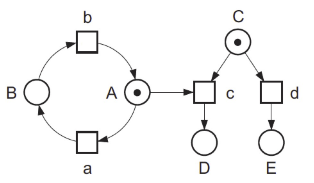
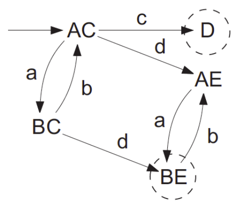


A sample elementary system net



... and its marking graph

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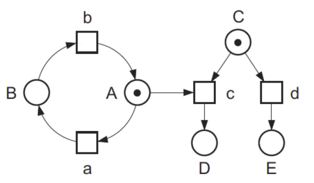

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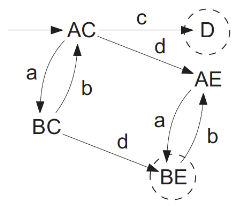


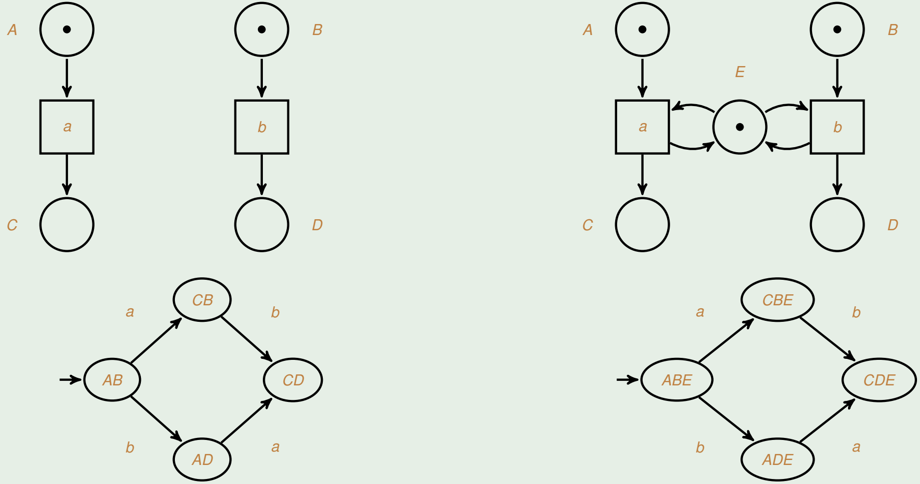
... and its marking graph

Interleaving semantics

The marking graph represents the interleaving semantics of a Petri net.

Interleaving vs. True Concurrency

Example 14.20 (Petri nets and their marking graphs)



Thus: Marking graphs are isomorphic even though the nets behave differently
(*a* and *b* can occur simultaneously on the left, but not on the right).

Outline of Lecture 14

Summary

- A Petri net consists of places, transitions and arcs.
- An elementary system net is a Petri net plus a marking.
- Firing a single transition in a marking is a step.
- A sequential run is a sequence of steps starting in the initial marking.
- The marking graph has as nodes the reachable markings of the net and as edges its reachable steps.
- The marking graph represents the interleaving semantics of a net.