

Concurrency Theory

Winter 2025/26

Lecture 6: Properties of Weak Bisimulation

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<https://proglang.github.io/teaching/25ws/ct.html>

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Outline of Lecture 6

- 1 Recap: Game Characterisation and Variants of Strong Bisimulation
- 2 Properties of Weak Bisimilarity
- 3 Weak Bisimilarity vs. Trace Equivalence
- 4 Deadlock Sensitivity
- 5 Congruence Property
- 6 Observation Congruence
- 7 Game Characterisation of Weak Bisimilarity

Rules of the Bisimulation Game

Rules

In each round, the current configuration (s, t) is changed as follows:

- (1) the **attacker** chooses one of the two processes in the current configuration, say t , and makes an $\xrightarrow{\alpha}$ -move for some $\alpha \in Act$ to t' , say, and
- (2) the **defender** must respond by making an $\xrightarrow{\alpha}$ -move in the other process s of the current configuration under the same action α , yielding $s \xrightarrow{\alpha} s'$.

The pair of processes (s', t') becomes the new current configuration.

The game continues with another round.

Results

- (1) If one player cannot move, the other player wins:
 - attacker cannot move if $s \not\rightarrow$ and $t \not\rightarrow$
 - defender cannot move if no matching transition available
- (2) If the game is played *ad infinitum*, the defender wins.

Game Characterisation of Bisimulation

Theorem (Game characterisation of bisimulation)

(Stirling 1995, Thomas 1993)

(1) $s \sim t$ iff *the defender has a universal winning strategy* from configuration (s, t) .

(2) $s \not\sim t$ iff *the attacker has a universal winning strategy* from configuration (s, t) .

(By means of a universal winning strategy, a player can always win, regardless of how the other player selects their moves.)

Proof.

by relating winning strategy of defender/attacker to existence/non-existence of strong bisimulation relation □

Thus, a bisimulation game can be used to prove bisimilarity as well as non-bisimilarity.¹ It often provides elegant arguments for $s \not\sim t$.

¹Later we will present yet another method to check this.

Strong Simulation

Observation: sometimes, the concept of strong bisimulation is **too strong** (example: extending a system by new features).

Definition (Strong simulation)

- Relation $\rho \subseteq \text{Prc} \times \text{Prc}$ is a **strong simulation** if, whenever $(P, Q) \in \rho$ and $P \xrightarrow{\alpha} P'$, there exists $Q' \in \text{Prc}$ such that $Q \xrightarrow{\alpha} Q'$ and $P' \rho Q'$.
- Q **strongly simulates** P , denoted $P \sqsubseteq Q$, if there exists a strong simulation ρ such that $P \rho Q$. Relation \sqsubseteq is called **strong similarity**.
- P and Q are **strongly simulation equivalent** if $P \sqsubseteq Q$ and $Q \sqsubseteq P$.

Thus: If Q strongly simulates P , then whatever transition P takes, Q can match it while retaining all of P 's options.

But: P does not need to be able to match each transition of Q !

Strong Simulation and Bisimilarity

Lemma (Bisimilarity implies simulation equivalence)

If $P \sim Q$, then $P \sqsubseteq Q$ and $Q \sqsubseteq P$.

Proof.

A strong bisimulation $\rho \subseteq \text{Proc} \times \text{Proc}$ for $P \sim Q$ is a strong simulation for both directions. □

Caveat: The converse does not generally hold!

Weak Bisimulation

Definition (Weak bisimulation)

(Milner 1989)

A binary relation $\rho \subseteq \text{Prc} \times \text{Prc}$ is a **weak bisimulation** whenever for every $(P, Q) \in \rho$ and $\alpha \in \text{Act}$ (including $\alpha = \tau$):

- (1) if $P \xrightarrow{\alpha} P'$, then there exists $Q' \in \text{Prc}$ such that $Q \xRightarrow{\alpha} Q'$ and $P' \rho Q'$, and
- (2) if $Q \xrightarrow{\alpha} Q'$, then there exists $P' \in \text{Prc}$ such that $P \xRightarrow{\alpha} P'$ and $P' \rho Q'$.

Definition (Weak bisimilarity)

Processes P and Q are **weakly bisimilar**, denoted $P \approx Q$, iff there is a weak bisimulation ρ with $P \rho Q$.

Thus,

$$\approx = \bigcup \{ \rho \subseteq \text{Prc} \times \text{Prc} \mid \rho \text{ is a weak bisimulation} \}.$$

Relation \approx is called **weak bisimilarity** or **observational equivalence**.

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Example 6.1 (A polling process)

(Koomen 1982)

A process that is willing to receive on port a or port b , and then terminates:

$$A? = a.\text{nil} + \tau.B?$$

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- Thus, \approx assumes that **if a process can escape from a τ -loop, it eventually will do so.**^a
Divergence (“livelock”) is a τ -loop.

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- Thus, \approx assumes that **if a process can escape from a τ -loop, it eventually will do so.**^a Divergence (“livelock”) is a τ -loop.
- Also note that $Div \approx nil$ where $Div = \tau.Div$.
- Thus, a **deadlocked process is weakly bisimilar to a process that can only diverge.**
- This is justified by the fact that “observations” can only be made by interacting with the process.

Example 6.2 (A simple communication protocol)

Observation: (unbounded) retransmission after communication failures can also cause τ -loops.

$$\begin{aligned} \text{Spec} &= \text{acc}.\overline{\text{del}}.\text{Spec}; \\ \text{Impl} &= (\text{Send} \parallel \text{Med} \parallel \text{Rec}) \setminus \{\text{send}, \text{trans}, \text{ack}, \text{error}\}; \\ \text{Send} &= \text{acc}.\text{Sending}; \\ \text{Sending} &= \overline{\text{send}}.\text{Wait}; \\ \text{Wait} &= \text{ack}.\text{Send} + \text{error}.\text{Sending}; \\ \text{Med} &= \text{send}.\text{Trans}; \\ \text{Trans} &= \overline{\text{trans}}.\text{Med} + \tau.\text{Err}; \\ \text{Err} &= \overline{\text{error}}.\text{Med}; \\ \text{Rec} &= \text{trans}.\text{Del}; \\ \text{Del} &= \overline{\text{del}}.\text{Ack}; \\ \text{Ack} &= \overline{\text{ack}}.\text{Rec}; \end{aligned}$$

(analyse using CAAL tool)

Properties of Weak Bisimilarity

Lemma 6.3 (Properties of \approx)

- (1) $P \sim Q$ implies $P \approx Q$.
- (2) \approx is an equivalence relation (i.e., reflexive, symmetric, and transitive).
- (3) \approx is the largest weak bisimulation.
- (4) \approx abstracts from τ -loops.

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Proof.

- (1) straightforward (as $\xrightarrow{\alpha} \subseteq \Longrightarrow^{\alpha}$)
- (2) similar to Lemma 4.7(1) for \sim
- (3) similar to Lemma 4.7(2) for \sim
- (4) see Examples 6.1 and 6.2



Lemma 6.4 (Milner's τ -laws)

$$\begin{aligned}\alpha.\tau.P &\approx \alpha.P \\ P + \tau.P &\approx \tau.P \\ \alpha.(P + \tau.Q) &\approx \alpha.(P + \tau.Q) + \alpha.Q\end{aligned}$$

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Proof.

by constructing appropriate weak bisimulation relations (left as an exercise)



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Weak Bisimilarity vs. Trace Equivalence

Definition 6.5 (Observational trace language)

The **observational trace language** of $P \in \text{Prc}$ is defined by:

$$\text{ObsTr}(P) := \{\hat{w} \in (A \cup \bar{A})^* \mid \text{there ex. } P' \in \text{Prc} : P \xrightarrow{w} P'\}$$

where \hat{w} is obtained from w by removing all τ -actions.

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Definition 6.6 (Observational trace equivalence)

$P, Q \in \text{Prc}$ are **observationally trace equivalent** if $\text{ObsTr}(P) = \text{ObsTr}(Q)$.

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$P, Q \in \text{Prc}$ are **observationally trace equivalent** if $\text{ObsTr}(P) = \text{ObsTr}(Q)$.

Theorem 6.7

$P \approx Q$ implies that P and Q are observationally trace equivalent. The reverse does not hold.

Proof.

similar to Theorem 4.9



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Observational Deadlocks

Definition 6.8 (Observational deadlock)

Let $P, Q \in \text{Prc}$ and $w \in \text{Act}^*$ such that

$P \xrightarrow{w} Q$ and there exists no $Q' \in \text{Prc}$ and $\lambda \in A \cup \bar{A}$ such that $Q \xRightarrow{\lambda} Q'$.

Then Q is called an **observational \hat{w} -deadlock** of P .

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Definition 6.9 (Observational deadlock sensitivity)

Relation $\equiv \subseteq \text{Prc} \times \text{Prc}$ is **observationally deadlock sensitive** whenever $P \equiv Q$ implies for every $v \in (A \cup \bar{A})^*$:

P has an observational v -deadlock iff Q has an observational v -deadlock.

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Theorem 6.10

\approx is observationally deadlock sensitive.

Proof.

similar to Theorem 4.13 for \sim



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Congruence

Lemma 6.11 (Partial CCS congruence property of \approx)

Whenever $P, Q \in \text{Prc}$ such that $P \approx Q$,

$$\alpha.P \approx \alpha.Q \quad \text{for every } \alpha \in \text{Act}$$

$$P \parallel R \approx Q \parallel R \quad \text{for every } R \in \text{Prc}$$

$$P \setminus L \approx Q \setminus L \quad \text{for every } L \subseteq A$$

$$P[f] \approx Q[f] \quad \text{for every } f : A \rightarrow A$$

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Proof.

omitted □

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Proof.

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What about choice?

- $\tau.a.\text{nil} \approx a.\text{nil}$ (cf. Example 5.11(1)) and $b.\text{nil} \approx b.\text{nil}$ (reflexivity).

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- But $\tau.a.\text{nil} + b.\text{nil} \not\approx a.\text{nil} + b.\text{nil}$. (Why?)
- Thus, weak bisimilarity is **not a CCS congruence**, which motivates a slight adaptation of \approx .

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Observation Congruence

Definition 6.12 (Observation congruence)

(Milner 1989)

$P, Q \in \text{Prc}$ are **observationally congruent**, denoted $P \approx^c Q$, if for every $\alpha \in \text{Act}$ (including $\alpha = \tau$):

- (1) if $P \xrightarrow{\alpha} P'$, then there is a sequence of transitions $Q \xRightarrow{\tau} \circ \xrightarrow{\alpha} \circ \xRightarrow{\tau} Q'$ such that $P' \approx Q'$ and
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Remark

- \approx^c differs from \approx only in that \approx^c requires τ -moves by P or Q to be mimicked by at least one τ -move in the other process.
- Moreover, this only applies to the **first step**; the successors just have to satisfy $P' \approx Q'$ (and not necessarily $P' \approx^c Q'$).

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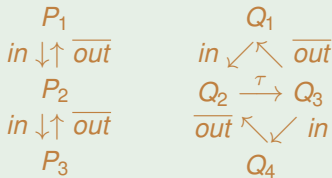
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- Moreover, this only applies to the **first step**; the successors just have to satisfy $P' \approx Q'$ (and not necessarily $P' \approx^c Q'$).
- Thus: if $P \not\xrightarrow{\tau}$ and $Q \not\xrightarrow{\tau}$, then $P \approx^c Q$ iff $P \approx Q$.

Example 6.13

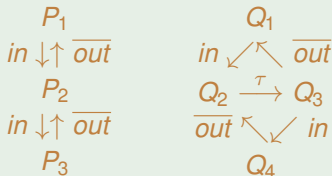
(1) Sequential and parallel two-place buffer:



$P_1 \approx^c Q_1$ since $P_1 \approx Q_1$ (cf. Example 5.11(3)) and neither P_1 nor Q_1 has initial τ -steps.

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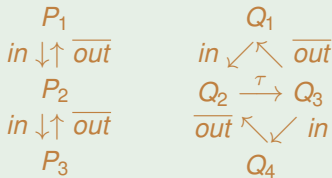


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(2) $\tau.a.nil \not\approx^c a.nil$ (since $\tau.a.nil \xrightarrow{\tau}$ but $a.nil \not\xrightarrow{\tau}$);
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(3) $a.\tau.nil \approx^c a.nil$ (since $\tau.nil \approx nil$).

Properties of Observation Congruence

Lemma 6.14

For every $P, Q \in \text{Prc}$,

- (1) $P \sim Q$ implies $P \approx^c Q$, and $P \approx^c Q$ implies $P \approx Q$
- (2) \approx^c is an equivalence relation
- (3) \approx^c is a CCS congruence
- (4) \approx^c is observationally deadlock-sensitive
- (5) $P \approx^c Q$ if and only if $P + R \approx Q + R$ for every $R \in \text{Prc}$
- (6) $P \approx Q$ if and only if ($P \approx^c Q$ or $P \approx^c \tau.Q$ or $\tau.P \approx^c Q$)

Proof.

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Proof.

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Note: (5) states that \approx^c is the “minimal fix” to establish congruence of \approx .

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Weak Bisimilarity as a Game

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In each round, the current configuration (s, t) is changed as follows:

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- (2) the **defender** must respond by making an $\xRightarrow{\alpha}$ -move in the other process s of the current configuration under the same action α , yielding $s \xRightarrow{\alpha} s'$.

The pair of processes (s', t') becomes the new current configuration.

The game continues with another round.

Weak Bisimilarity as a Game

Rules

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The game continues with another round.

Results

- (1) If one player cannot move, the other player wins.
 - attacker cannot move if $s \not\rightarrow$ and $t \not\rightarrow$
 - defender cannot move if no matching transition available
- (2) If the game can be played *ad infinitum*, the defender wins.

Game Characterisation of Weak Bisimilarity

Theorem 6.15 (Game characterisation of weak bisimilarity) (Stirling 1995, Thomas 1993)

(1) $s \approx t$ iff *the defender has a universal winning strategy* from configuration (s, t) .

(2) $s \not\approx t$ iff *the attacker has a universal winning strategy* from configuration (s, t) .

(By means of a universal winning strategy, a player can always win, regardless of how the other player selects their moves.)

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Proof.

by relating winning strategy of defender/attacker to existence/non-existence of weak bisimulation relation □